High energy string-brane scattering for massive states

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Abstract

String-brane interactions provide an ideal framework to study the dynamics of the massive states of the string spectrum in a non-trivial background. We present here an analysis of tree-level amplitudes for processes in which an NS-NS string state from the leading Regge trajectory scatters from a D-brane into another state from the leading Regge trajectory, in general of a different mass, at high energies and small scattering angles. This is done by using world-sheet OPE methods and effective vertex operators. We find that this class of processes has a universal dependence on the energy of the projectile. We then compare the result for these inelastic processes with that which one would obtain from the eikonal operator in a non-trivial test of its ability to describe transitions between different string mass levels. The two are found to be in agreement.

1 Introduction

In its current state string theory provides us with a perturbative description of quantum gravity. It is unclear whether this description is valid in all regimes, or what the fundamental degrees of freedom might be, yet it is clear that it can be put to good use in learning about physics at the Planck scale where we expect gravitational interactions to become significant. Such insight may help us understand processes such as the formation of black holes, and their absorption of matter. Furthermore, by examining high energy scattering one may test the mathematical fortitude of string theory – its ability to yield unitary amplitudes and finite quantum corrections – as well as probing its short-distance structure.

Early investigations into the high energy behaviour of scattering amplitudes in string theory examined two principal regimes, that of fixed scattering angle [1–3] and that of fixed momentum transfer [4–6]. In the former it was discovered that at each order in the perturbative expansion the high energy behaviour of all string scattering amplitudes is dominated by a saddle point in the moduli space of the corresponding Riemann surfaces. As a result one could interpret these processes in terms of a classical string trajectory in spacetime. However, in the limit for which these results are valid one encounters difficulties with the convergence of the perturbative expansion [7] and application of these results is potentially problematic. The latter investigations consider graviton-graviton scattering in the limit of large centre of mass energy and fixed momentum transfer, referred to as the Regge limit; these revealed some interesting features arising at different values of the transferred momentum t. For small t, corresponding to large distance processes, it was found that elastic scattering dominates and this is mediated by the long range exchange of gravitons. For larger values of t one encounters a semiclassical eikonal description which reveals effects attributable to classical gravity, that is to say, the strings begin to feel the influence of an effective curved background. If the value of t is increased further absorptive processes begin to contribute significantly due to the production of inelastic and diffractive states and beyond this the analysis can be extended into the regime of small but fixed scattering angle.

More recently there has been renewed interest in the high energy behaviour of scattering amplitudes in string theory [8–10]. In [10] the methods employed in [5] are used to examine the high energy scattering of a graviton from a stack of N parallel Dp-branes in a flat background. Significantly, this process involves the interaction between a perturbative and non-perturbative object for which we have a microscopic description, whereas previously only interactions between perturbative states had been investigated. As in the earlier investigations the limit of large energies is examined in a number of kinematic regions for which both the impact parameter and scale of curvature* produced by the D-branes are larger than the string length. For the largest impact parameters one finds a region of perturbative scattering leading to vanishingly small de-

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*by which we mean R_p, see equation (19)

flection angles, and as the impact parameter is reduced one encounters both classical corrections and corrections due to the nonzero string length. As the impact parameter approaches the scale of curvature one expects to see gravitational effects and the string can be considered classically to have been captured; it has been suggested that this would be an interesting region for further study, together with impact parameters much smaller than the curvature, since an understanding of these regions could provide one with the tools required to study the microscopic process of matter falling past a horizon.

In this paper we are motivated by these previous works to study the interactions of a massive closed superstring with a stack of Dp-branes. Such interactions will in general include inelastic processes in which the string is excited/decays into a state of a different mass. Since we expect scattering at large energies to be particularly simple for the leading Regge trajectory – those states in the string spectrum with the maximum spin possible for their mass – we will take such states as our two external strings. The presence of the D-branes will break the $SO(1,9)$ invariance to $SO(1,p) \times SO(9-p)$ and momentum will only be conserved parallel to the world volume of the D-branes; as such, the invariant quantities we will primarily consider here shall be composed of the momentum flowing parallel to the D-branes and the momentum transferred to the D-branes and after a suitable transformation the former becomes the energy of the projectile. We will be interested in the limit in which this energy becomes extremely large in comparison to the string scale while the magnitude of the momentum exchanged is kept fixed. In order to obtain high energy scattering amplitudes in the most efficient manner we will make use of operator product expansion (OPE) methods to construct effective vertex operators as pioneered in [11, 12] and more recently used in [13, 14]. Insertion of these effective vertices onto the upper-half of the complex plane will yield the tree-level amplitudes of these processes which show identical Regge behaviour to that exhibited in the graviton-graviton scattering. This behaviour is universal for the states on the leading Regge trajectory. An analysis using the eikonal operator of [10] gives results in agreement with those produced by world sheet calculations, providing further verification of its capability in computing elements of the string S-matrix. This is a non-trivial confirmation of the efficacy of the eikonal operator in analysing scattering beyond the case of massless states. Furthermore, these checks demonstrate by the explicit calculation of string amplitudes that longitudinal excitations are absent in the scattering of a massive string with a D-brane for large impact parameters, as indicated in [10] by computations concerning the quantisation of a string in the background generated by the D-branes.

The contents of this paper are as follows. In Section 2 we lay out our conventions for the kinematics of string-brane scattering and describe the particular regions of the parameter space in which we are interested. In Section 3 we introduce the string states that we will use, that is, the states of the NS-NS sector in type II string theory with maximal spin in comparison to other states with the same mass; these states form the leading Regge trajectory. As an example it is shown how to determine the BRST invariant vertex operator for the first massive state on the

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leading Regge trajectory, and this result is then generalised to give the vertex operator for a state with mass $\alpha' M^2 = 4n$. In Section 4 we consider the amplitude for the inelastic excitation of a state with $n = 0$ to a state with $n' = 1$ and study its high energy limit. This direct evaluation of the amplitudes can then be compared to the results of Section 5 in which we construct an effective vertex using OPE methods in order to compute the leading high energy contribution for a generic inelastic process involving states of the leading Regge trajectory. In Section 6 we show that in the Regge limit the tree-level string amplitudes are precisely reproduced by the matrix elements of the eikonal operator introduced in [10] and hence briefly examine what this may imply for the scattering of massive states at large impact parameters. In Section 7 we discuss these results and their implications for other high energy processes. In particular we comment on the interesting limit of very large masses for the external states.

2 Kinematics of a string scattering from a Dp-brane

Here we briefly review the kinematics appropriate to tree-level interactions between a single string and a Dp-brane. Throughout this work we will use a flat metric of positive signature, $\eta = diag(-, +, +, ..., +)$. In the following computations we will consider some initial state with momentum p_1 such that $\alpha' M_1^2 = -\alpha' p_1^2 = 4n$; after interacting with the Dp-brane we are left with another state with momentum p_2 which in general can have a different mass, $\alpha' M_2^2 = -\alpha' p_2^2 = 4n'$. We can decompose these momenta into vectors which are parallel and orthogonal to the directions in which the D-brane is extended, that is

$$p_i^\mu = p_{i\parallel}^\mu + p_{i\perp}^\mu. \tag{1}$$

The mass of a D-brane scales as $1/g_s$ and so to leading order in the perturbative expansion it is infinitely massive. Consequently we may neglect its recoil and momentum is only conserved in directions parallel to the D-brane,

$$p_{1\parallel}^\mu + p_{2\parallel}^\mu = 0. \tag{2}$$

For brevity we denote $p_{1\parallel}^\mu = -p_{2\parallel}^\mu = p_\parallel^\mu$. Using the mass-shell condition we may relate this quantity to the orthogonal components of momentum

$$p_{1\perp}^2 = -p_\parallel^2 - \frac{4n}{\alpha'}, \tag{3a}$$

$$p_{2\perp}^2 = -p_\parallel^2 - \frac{4n'}{\alpha'}. \tag{3b}$$

The vectors $p_{i\perp}$ are by definition space-like[†] so we can infer that $p_\parallel$ is necessarily time-like. Considering this we define the following kinematic invariants for use as the parameters in our

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[†]ignoring the specific case of a D-instanton.

computations

$$s \equiv -p_\parallel^2, \tag{4a}$$

$$t \equiv -(p_1 + p_2)^2, \tag{4b}$$

$$E \equiv \sqrt{s}. \tag{4c}$$

After a Lorentz transformation to a frame in which the spatial components of p_1 are nonzero only in the orthogonal directions the quantity E will be equal to the energy of the string.

By definition, s and t have the following physical boundaries,

$$\max\{M_1^2,\ M_2^2\} \leq s < \infty, \tag{5}$$

$$\left(\sqrt{s - M_1^2} - \sqrt{s - M_2^2}\right)^2 \leq |t| \leq \left(\sqrt{s - M_1^2} + \sqrt{s - M_2^2}\right)^2. \tag{6}$$

We could explore various kinematic regimes, depending on the relative sizes of the initial and final mass of the string, the square of the string energy s and the momentum transfer t. In this paper we will analyse the Regge regime of such amplitudes, meaning their behaviour for $\alpha's \to \infty$ and $\alpha't$ fixed. We will see that for massive string states from the leading Regge trajectory these amplitudes demonstrate typical Regge behaviour. More precisely, we will keep the external states fixed while taking the large s limit, which implies that the kinetic energy of the projectile is very large.

As we will show in section 5, if we consider Regge kinematics the leading contribution to the amplitudes is captured by the OPE of the vertex operators inserted far from the boundary of the world-sheet. The amplitudes could be dominated by the OPE and show Regge behaviour at high energy also when we consider external states with very large masses, as long as the difference $n' - n$ is negligible compared to $\alpha's$ so that t/s remains small. If instead we allow a large mass gap between initial and final states and we let it grow with the projectile energy, $\Delta M \sim E$, then we are in a kinematic regime where $t \sim s$. In this regime we expect the leading contribution to the amplitude to be given not by the OPE but by a semiclassical world-sheet corresponding to a saddle-point in the moduli space. The amplitudes should then show an exponential decay in the energy, typical of scattering processes at fixed angle [1–3, 15]. The analysis of the scattering amplitudes in the limit of large masses for the external states is however beyond the scope of this paper.

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3 Massive spectrum of the NS-NS superstring on the leading Regge trajectory

In this section we analyze the massive spectrum of the NS-NS sector of the type II superstring, focusing on the highest spin states at a given mass, i.e, the leading Regge trajectory. As an illustrative example, we will review the BRST quantization of the first level, following the conventions used in [16, 17].

The mass shell condition for the nth level of the NS-NS sector of the superstring after the GSO projection is given by

$$\alpha' M^2 = 4n, \tag{7}$$

with $n = 0, 1, 2...$. In the following we will write the closed string state as the product of two copies of the open string sector, so the states we will consider carry momentum $p^\mu = 2k^\mu$ where k will represent the momentum of the open string vertex. Henceforth, when a result is stated for holomorphic fields, it is with the understanding that an analogous result will hold for the antiholomorphic quantities.

For the first massive level the mass shell condition (7) with $n = 1$ is satisfied by the following states in the -1 picture:

$$|\phi_\varepsilon\rangle = \varepsilon_{\mu\nu} \alpha^\mu_{-1} \psi^\nu_{-\frac{1}{2}} |0; k\rangle, \tag{8a}$$

$$|\phi_A\rangle = \mathcal{A}_{\mu\nu\rho} \psi^\mu_{-\frac{1}{2}} \psi^\nu_{-\frac{1}{2}} \psi^\rho_{-\frac{1}{2}} |0; k\rangle, \tag{8b}$$

$$|\phi_B\rangle = B_\mu \psi^\mu_{-\frac{3}{2}} |0; k\rangle. \tag{8c}$$

To find the physical states, we write the most general state at this level as a linear combination:

$$|\phi\rangle = \left(\varepsilon_{\mu\nu} \alpha^\mu_{-1} \psi^\nu_{-\frac{1}{2}} + \mathcal{A}_{\mu\nu\rho} \psi^\mu_{-\frac{1}{2}} \psi^\nu_{-\frac{1}{2}} \psi^\rho_{-\frac{1}{2}} + B_\nu \psi^\nu_{-\frac{3}{2}} \right) |0; k\rangle. \tag{9}$$

The corresponding vertex operator with superghost charge -1 is

$$V_{-1} = c e^{-\varphi} \left(\frac{i}{\sqrt{2\alpha'}} \varepsilon_{\mu\nu} \partial X^\mu \psi^\nu + \mathcal{A}_{\mu\nu\rho} \psi^\mu \psi^\nu \psi^\rho + B_\nu \partial \psi^\nu \right) e^{ik \cdot X}, \tag{10}$$

where the field c is one of the reparametrisation ghosts and the field φ, together with the η and ξ fields gives the bosonisation of the superghosts. In order for this state to be physical it must be invariant under BRST transformations.

As shown in [18], we can gauge away the scalar, the antisymmetric rank-2 tensor and the vector. The requirement of BRST invariance for the remaining states implies the constraints

$$k^\mu \epsilon_{(\mu\nu)} = 0, \quad k^\mu \mathcal{A}_{[\mu\nu\rho]} = 0. \tag{11}$$

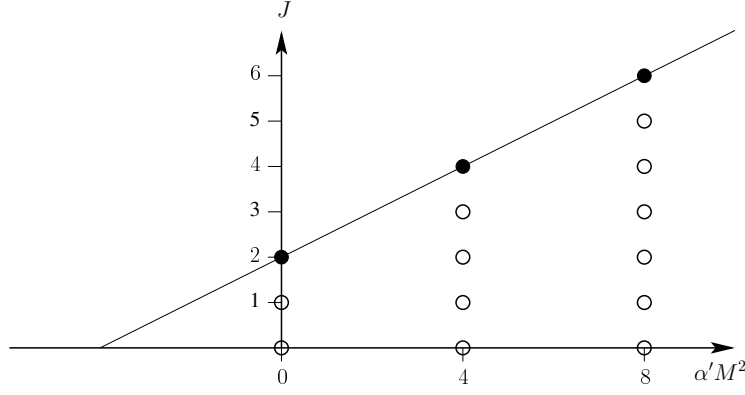We are left with two physical states:

Figure 1: Diagrammatic depiction of the type II string spectrum indicating the mass and spin of each physical state, represented by circles. Filled circles show states on the leading Regge trajectory.

- The state $\varepsilon_{(\mu\nu)} \alpha_{-1}^\mu \psi_{-\frac{1}{2}}^\nu |0;0\rangle$ has a polarization $\varepsilon_{(\mu\nu)}$ which is a completely symmetric, traceless tensor invariant under the litte group $SO(9)$, so it carries 44 degrees of freedom.

- The state $\mathcal{A}_{[\mu\nu\rho]} \psi_{-\frac{1}{2}}^\mu \psi_{-\frac{1}{2}}^\nu \psi_{-\frac{1}{2}}^\rho$ has a polarization $\mathcal{A}_{[\mu\nu\rho]}$ which is a three-form of $SO(9)$, corresponding to 84 degrees of freedom.

Together, these two states have 128 degrees of freedom, which is the full bosonic content of the left sector of the first massive level, as explained in [19].

As we consider higher and higher levels, it can be a difficult task to find the BRST invariant vertex operators, since the number of states increases dramatically. Nevertheless, at each level one could consider the states in the light-cone gauge, and these can always be rearranged into irreducible representations of the little group of $SO(9)$, which describe the BRST-invariant states. This has been shown in [18] for the first two levels.

In this paper we shall focus on states belonging to the leading Regge trajectory. The corresponding vertex operators have a particularly simple form. We first construct the vertex for an open string state which is totally symmetric in its polarisation. In general the tensor product of two such states will give a closed string state in a reducible representation containing physical states of the same mass but different spin. However, by symmetrising over all indices of the polarisation one is left with a state of spin $J = (\alpha' M^2 + 4)/2$. This is the maximum possible spin in the type II string spectrum for a fixed mass level and it is the set of all such states that comprises the leading Regge trajectory. This can be seen pictorially in figure 1. Written in terms of oscillators, such an open string state will have the form $|\phi_n\rangle = \varepsilon_{\mu_1 \ldots \mu_n \alpha} \prod_{i=1}^n \alpha_{-1}^{\mu_i} \psi_{-\frac{1}{2}}^\alpha |0;k\rangle$,

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and this will give the following closed string state,

$$|\phi_n\rangle^{closed} = \epsilon_{\mu_1...\mu_n \, \nu_1...\nu_n(\alpha\beta)} \left[\prod_{i=1}^{n} \alpha^{\mu_i}_{-1} \widetilde{\alpha}^{\nu_i}_{-1} \right] \psi^\alpha_{-\frac{1}{2}} \widetilde{\psi}^\beta_{-\frac{1}{2}} |0;p\rangle , \tag{12}$$

where $\epsilon_{\mu_1...\mu_n\alpha\nu_1...n_n\beta} = \varepsilon_{\mu_1...\mu_n\alpha} \otimes \widetilde{\varepsilon}_{\nu_1...\nu_n\beta}$. From here on in it shall be implicitly understood that the polarisation ϵ is symmetric in all indices.

By virtue of the state-operator isomorphism we obtain from the state above a vertex operator of superghost charge (-1,-1),

$$W^{(n)}_{(-1,-1)}(k,z,\bar{z}) = \epsilon_{\mu_1...\mu_n\alpha \, \nu_1...\nu_n\beta} V^{\mu_1...\mu_n\alpha}_{-1}(k,z) \widetilde{V}^{\nu_1...\nu_n\beta}_{-1}(k,\bar{z}), \tag{13}$$

with

$$V^{\mu_1...\mu_n\alpha}_{-1}(k,z) = \frac{1}{\sqrt{n!}} \left(\frac{i}{\sqrt{2\alpha'}} \right)^n e^{-\varphi(z)} \left(\prod_{i=1}^{n} \partial X^{\mu_i} \right) \psi^\alpha e^{ik\cdot X(z)}. \tag{14}$$

Above we have written $X(z)$ for the open string field, obtained by decomposing the complete string coordinate field into $X(z,\bar{z}) = (X(z) + \widetilde{X}(\bar{z}))/2$, and as previously stated we make use of the open string momentum k for convenience while the physical momentum of the string is $p = 2k$. Since we wish to compute amplitudes on a world-sheet with the topology of the disc the vertices which make up this amplitude should have a total superghost charge of -2, consequently we will need the vertex operators of the state above in the $(0,0)$ picture,

$$W^{(n)}_{(0,0)}(k,z,\bar{z}) = -\epsilon_{\mu_1...\mu_n\alpha \, \nu_1...\nu_n\beta} V^{\mu_1...\mu_n\alpha}_{0}(k,z) \widetilde{V}^{\nu_1...\nu_n\beta}_{0}(k,\bar{z}) \tag{15}$$

where

$$V^{\mu_1...\mu_n\alpha}_{0}(k,z) = \frac{1}{\sqrt{n!}} \left(\frac{i}{\sqrt{2\alpha'}} \right)^{n+1} (\partial X^{\mu_n} \partial X^\alpha - 2i\alpha' k \cdot \psi \partial X^{\mu_n} \psi^\alpha - 2\alpha' n \partial \psi^{\mu_n} \psi^\alpha)$$

$$\left(\prod_{i=1}^{n-1} \partial X^{\mu_i} \right) e^{ik\cdot X(z)}. \tag{16}$$

Before moving on we should make one point; strictly speaking, the vertices given in equations (16) and (14) can only be applied to the cases $n > 0$, however we shall extend their use to the case $n = 0$ with the understanding that when one encounters a product of the form $\prod_{i=1}^{0} a_i$ we are to replace it by unity.

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4 Computation of the amplitudes on the disk in the high energy limit

The scattering amplitude for the interaction of two closed string states with a stack of N Dp-branes at tree level is given by the insertion of two closed string vertex operators onto the upper-half of the complex plane,

$$A_{n,n'} = \mathcal{N} \int_{\mathbb{H}_+} \frac{dz_1^2 dz_2^2}{V_{CKG}} \left\langle W_{(0,0)}^{(n)}(k_1, z_1, \bar{z}_1) W_{(-1,-1)}^{(n')}(k_2, z_2, \bar{z}_2) \right\rangle_{\mathbb{H}_+}. \tag{17}$$

Here we have the vertex operator (15) for a state with momentum $p_1 = 2k_1$ and mass $M^2 = 4n/\alpha'$ carrying a superghost charge $(0,0)$ and the vertex operator (13) for a state with momentum $p_2 = 2k_2$ and mass $M^2 = 4n'/\alpha'$ carrying a superghost charge $(-1,-1)$. In this section we will compute the amplitude involving one graviton and one massive symmetric state at the level $n' = 1$. In doing so we shall see many features which are not only common to the methods introduced in Section 5 but also motivate them.

The normalization constant, $\mathcal{N}$, in (17) is formed from the product of the normalizations of the vertices, $(\kappa/2\pi)^2$, and the topological factor for a disc amplitude $C_{D_2} = 2\pi^2 T_p/\kappa$, where κ is the gravitational coupling constant in ten dimensions and T_p is the coupling for closed string states to a Dp-brane. Overall this gives the normalization

$$\mathcal{N} = \frac{\kappa T_p}{2} = \frac{R_p^{7-p} \pi^{\frac{9-p}{2}}}{\Gamma\left(\frac{7-p}{2}\right)}, \tag{18}$$

where R_p represents a characteristic size for the stack of D-branes and is related to the t'Hooft coupling, $\lambda = gN$ as follows,

$$R_p^{7-p} = gN \frac{(2\pi\sqrt{\alpha'})^{7-p}}{(7-p)\Omega_{8-p}}, \qquad \Omega_n = \frac{2\pi^{\frac{n+1}{2}}}{\Gamma\left(\frac{n+1}{2}\right)}. \tag{19}$$

Hence, using equations (14–15) it is easily verified that the vertex for the graviton in the $(0,0)$ picture is

$$W_{(0,0)}^{(0)}(k_1, z_1, \bar{z}_1) = -\epsilon_{\mu\nu} V_0^\mu(k_1, z_1) \widetilde{V}_0^\nu(k_1, \bar{z}_1) \tag{20}$$

with

$$V_0^\mu(k_1, z_1) = \frac{1}{\sqrt{2\alpha'}} \left(i\partial X^\mu(z_1) + 2\alpha' k_1 \cdot \psi(z_1)\psi^\mu(z_1) \right) e^{ik_1 \cdot X(z_1)}, \tag{21}$$

and the vertex operator in the $(-1,-1)$ picture for the state at the level $n = 1$ is

$$W_{(-1,-1)}^{(1)}(k_2, z_2, \bar{z}_2) = G_{\rho\sigma\tau\zeta} V_{-1}^{\rho\sigma}(k_2, z_2) \widetilde{V}_{-1}^{\tau\zeta}(k_2, \bar{z}_2) \tag{22}$$

with

$$V_{-1}^{\rho\sigma}(k_2, z_2) = \frac{i}{\sqrt{2\alpha'}} e^{-\varphi(z_2)} \partial X^\rho(z_2) \psi^\sigma(z_2) e^{ik_2 \cdot X(z_2)}. \tag{23}$$

We are interested in a scattering process in the presence of D-branes which break Lorentz invariance; this changes the kinematics and the boundary conditions (Neumann on the direction parallel to the brane, Dirichlet on the directions orthogonal to it). To implement these new conditions, we use the doubling trick. As reviewed in [20] in the context of D-brane physics, this simiplifies the treatment by substituting antiholomorphic fields $\bar{X}(\bar{z})$, $\tilde{\varphi}(\bar{z})$ which are functions of an antiholomorphic variable, with holomorphic fields depending on $\bar{z}$ treated as an independent holomorphic variable. This is equivalent to sending $X^\mu(\bar{z}) \to D^\mu{}_\nu X^\nu(\bar{z})$, $\tilde{\psi}^\mu(\bar{z}) \to D^\mu{}_\nu \psi^\nu(\bar{z})$ and $\tilde{\varphi}(\bar{z}) \to \varphi(\bar{z})$ for correlation functions evaluated on $\mathbb{H}_+$, where $D = (\eta_{p+1}, -1_{9-p})$. Having employed the doubling trick we need only compute correlators between the holomorphic fields. We will use

$$\langle X^\mu(z) X^\nu(w) \rangle = -2\alpha' \eta^{\mu\nu} \log(z - w),$$
$$\langle \psi^\mu(z) \psi^\nu(w) \rangle = \frac{\eta^{\mu\nu}}{z - w},$$
$$\langle \phi(z) \phi(w) \rangle = -\log(z - w). \tag{24}$$

There are several kinematic factors which appear in the following calculation which depend on the momenta carried by the holomorphic and antiholomorphic fields; we would like to be able to express these in terms of the variables above. From the definition of $D^\mu{}_\nu$ it can be seen that the momenta satisfy $(D \cdot k)^\mu = k_\parallel^\mu - k_\perp^\mu$ and so, with the aid of the conservation of momentum, one can deduce the following identities,

$$k_1 \cdot k_2 = \frac{n}{2\alpha'} + \frac{n'}{2\alpha'} - \frac{t}{8}, \tag{25}$$

$$k_1 \cdot D \cdot k_1 = -\frac{s}{2} + \frac{n}{\alpha'}, \tag{26}$$

$$k_2 \cdot D \cdot k_2 = -\frac{s}{2} + \frac{n'}{\alpha'}, \tag{27}$$

$$k_1 \cdot D \cdot k_2 = \frac{s}{2} + \frac{t}{8} - \frac{n}{2\alpha'} - \frac{n'}{2\alpha'}. \tag{28}$$

Inserting the vertex operators (20-23) and the normalization (18) into the integral (17) we

obtain

$$
\begin{aligned}
A_{0,1} =& \frac{\kappa T_p}{2} \int_{\mathbb{H}_+} \frac{d^2 z_1 d^2 z_2}{V_{CKG}} \left\langle W^{(0)}_{(0,0)}(k_1, z_1, \bar{z}_1) W^{(1)}_{(-1,-1)}(k_2, z_2, \bar{z}_2) \right\rangle_{\mathbb{H}_+} \\
=& \frac{\kappa T_p}{8\alpha'^2} \, \epsilon_{\mu\lambda} D^\lambda \,_\nu G_{\rho\sigma\alpha\beta} D^\alpha \,_\tau D^\beta \,_\zeta \int_{\mathbb{H}_+} \frac{d^2 z_1 d^2 z_2}{V_{CKG}} \\
& \left\langle \, : \left(i\partial X^\mu(z_1) + 2\alpha' k_1 \cdot \psi(z_1) \psi^\mu(z_1) \right) e^{ik_1 \cdot X(z_1)} : \right. \\
& \quad : \left(i\bar{\partial} X^\nu(\bar{z}_1) + 2\alpha' k_1 \cdot D \cdot \psi(\bar{z}_1) \psi^\nu(\bar{z}_1) \right) e^{ik_1 \cdot D \cdot X(\bar{z}_1)} : \\
& \quad \left. : e^{-\varphi(z_2)} \partial X^\rho(z_2) \psi^\sigma(z_2) e^{ik_2 \cdot X(z_2)} :: e^{-\varphi(\bar{z}_2)} \bar{\partial} X^\tau(\bar{z}_2) \psi^\zeta(\bar{z}_2) e^{ik_2 \cdot D \cdot X(\bar{z}_2)} : \, \right\rangle.
\end{aligned}
$$

(29)

In evaluating the above correlator it can be shown that the leading term in the high energy Regge limit is given by the contraction of the two operators quadratic in the fermionic fields. This term is proportional to $2\alpha' \, k_1 \cdot D \cdot k_1 = \alpha' s + 2$ and the overall s factor assures that this term is dominant with respect to all the other contractions in the amplitude.

In order to see that this is the case it is important to note the following fact. If we define an $SL(2, \mathbb{R})$ invariant variable

$$
\omega = \frac{(z_1 - z_2)(\bar{z}_1 - \bar{z}_2)}{(z_1 - \bar{z}_2)(\bar{z}_1 - z_2)}
$$

(30)

and use this in writing the factor in the correlation function which results from the contraction of the $e^{ik \cdot X}$ operators

$$
\left\langle e^{ik_1 \cdot X(z_1)} e^{ik_1 \cdot D \cdot X(\bar{z}_1)} e^{ik_2 \cdot X(z_2)} e^{ik_2 \cdot D \cdot X(\bar{z}_2)} \right\rangle = \omega^{-\alpha' \frac{t}{4} + 1} (\omega - 1)^{-\alpha' s} (z_2 - \bar{z}_2)^2,
$$

(31)

then the remaining explicit z-dependence can be combined with that from the other possible contractions in (29), together with the appropriate measure $\frac{d^2 z_1 d^2 z_2}{V_{CKG}} \mapsto d\omega (z_1 - \bar{z}_2)^2 (\bar{z}_1 - z_2)^2$, to give some multiplicative $SL(2, \mathbb{R})$ invariant function $F(\omega)$. The amplitude then takes the following schematic form

$$
A_{0,1} = \frac{\kappa T_p}{2} \int_0^1 d\omega \, \omega^{-\alpha' \frac{t}{4} + 1} (\omega - 1)^{-\alpha' s} F(\omega).
$$

(32)

The behaviour of this integral when we take the limit $\alpha' s \to \infty$ is controlled by the integrand in the neighbourhood of the point $\omega = 0$ and since $F(\omega)$ is, in general, a sum of terms composed of powers of ω, $(1 - \omega)$ and their inverse quantities, $A_{0,1}$ itself consists of a sum of integrals of the form shown below,

$$
\int_0^1 d\omega \, \omega^{-\alpha' \frac{t}{4} + a} (1 - \omega)^{-\alpha' s + b} = \frac{\Gamma\left(-\alpha' \frac{t}{4} + a + 1 \right) \Gamma\left(-\alpha' s + b + 1 \right)}{\Gamma\left(-\alpha' \frac{t}{4} - \alpha' s + a + b + 2 \right)}.
$$

(33)

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It can be seen that in the high energy limit this quantity will scale with s as $(\alpha' s)^{-a-1}$ and therefore the dominant contribution to $A_{0,1}$ will come from the term with the lowest value of a. The definition of ω in equation (30) implies that these both point to large s behaviour being governed by the region of integration over the world sheet in which the two vertex operators are brought together, that is to say when $z_1 \to z_2$ and $\bar{z}_1 \to \bar{z}_2$. The analysis of this process will be expanded upon and made more systematic in the next Section.

Having learnt this, one can quickly deduce that we are interested in the terms of $F(\omega)$ which are obtained from the maximum possible number of contractions between the holomorphic and antiholomorphic fields in equation (29). In general there are many such terms but, as mentioned below (29), those which contain the contraction $\overline{k_1 \cdot \psi \, k_1 \cdot D \cdot \psi}$ will bring an additional factor $\alpha' s$ and so will ultimately be the leading terms at high energy.

By evaluating the correlation function in equation (29), then employing the physical state conditions $\epsilon_{\mu\nu} k_1^\mu = G_{\rho\sigma\tau\zeta} k_2^\rho = 0$ and momentum conservation $k_1^\mu + (D \cdot k_1)^\mu + k_2^\mu + (D \cdot k_2)^\mu = 0$, we can determine the function $F(\omega)$ and perform the integral in equation (32). If we take the limit $\alpha' s \to \infty$ we find that only one term dominates in this case, which is proportional to $k_1^\rho G_{\rho\sigma\tau\zeta} \, \epsilon^{\sigma\tau} k_1^\zeta$. Subsequent application of Stirling's approximation for the Gamma function yields the following form for the amplitude in the high energy limit

$$A_{0,1} \sim \frac{\kappa T_p}{2} e^{-i\pi\alpha' \frac{t}{4}} \Gamma\left(-\frac{\alpha' t}{4}\right) (\alpha' s)^{\frac{\alpha' t}{4}+1} \frac{\alpha'}{2} q^\rho G_{\rho\sigma\tau\zeta} \, \epsilon^{\sigma\tau} q^\zeta, \tag{34}$$

where we have replaced k_1 and k_2 with the physical momentum transferred $q = 2(k_1 + k_2)$ by virtue of the physical state conditions.

It is instructive to compare this result with the analogous result for the elastic scattering of a graviton by a D-brane,

$$A_{0,0} \sim \frac{\kappa T_p}{2} e^{-i\pi\alpha' \frac{t}{4}} \Gamma\left(-\frac{\alpha' t}{4}\right) (\alpha' s)^{\frac{\alpha' t}{4}+1} \epsilon_{1\sigma\tau} \, \epsilon_2^{\sigma\tau}. \tag{35}$$

Considered purely as functions of the complex variables s and t, these two equations demonstrate identical behaviour; the physical amplitude is obtained by taking s, t real with large positive s and negative t, but for positive t we see that there are poles corresponding to the exchange of a string of mass $M^2 = t$ with the D-brane and, futhermore, the s-dependence indicates that the exchanged string has spin $J = (\alpha' M^2 + 4)/2$ — it belongs to the Regge trajectory of the graviton. As the energy becomes very large this description of the exchange of a single string breaks down due to the violation of unitarity, however, as demonstrated in [10] unitarity may be recovered by taking into account loop effects. Examining the two amplitudes above we note that the elastic and the inelastic amplitude only differ in the multiplicative factor containing the initial and final polarisation tensors and the exchanged momentum.

In the next Section we shall generalise this result and compute the leading high energy contribution to scattering amplitudes involving two generic states of the leading Regge trajectory, at mass levels n and n' of the spectrum; to do this we shall make use of the OPE for the two vertex operators to isolate those terms in the amplitude which dominate as the vertices are brought together on the world sheet.

5 Computation via OPE methods

It is seen in section 4 that the usual methods used for calculating scattering amplitudes in string theory can result in the generation of a vast number of terms which transpire to be subleading in energy after integration over the world-sheet. If our interests lie only in the leading terms then we would like to be able to distinguish these subleading terms and discard them at the very beginning of our calculation; with OPE methods we can do this quite easily and, furthermore, they highlight the simple structure present in the class of amplitudes we consider here.

To compute the leading terms in the amplitude given by equation (17) in the Regge limit, $\alpha' s \to \infty$ and $\alpha' t$ fixed, it is key to note that the integral over the world-sheet in (17) is dominated at large $\alpha' s$ by the behaviour of the vertex operators as $z_1 \to z_2$, as exemplified in equation (32). Specifically, when working in the Regge regime for scattering processes with two external string states the leading contribution to the integral over the world-sheet can be taken from the OPE of the vertex operators. Subsequent integration over their separation $w = z_1 - z_2$ will give an amplitude with the expected Regge behaviour; however, care must be taken in this process, as we will see, since there exist terms subleading in w which will contribute factors of $\alpha' s$ and by doing so prevent us from neglecting them.

Rather than evaluating the correlator in (17), integrating the result and then computing the asymptotic form for large $\alpha' s$ we may instead determine the OPE of the two vertex operators and integrate out the dependence on the separation $w = z_1 - z_2$. This process yields a quasi-local operator referred to as the pomeron vertex operator, first introduced in [11] as the reggeon and recently used in [13, 14]. As illustrated in figure 2(a), the process of constructing this operator involves taking the limit in which two physical vertices approach one another on a surface topologically equivalent to the infinite cylinder, hence it can be considered to properly describe the t-channel exchange of a closed string. The pomeron vertex operator itself, up to subleading term in s, also satisfies the physical state conditions for any $\alpha' t$, and for $\alpha' t = 4n$ ($n = 0, 1, \ldots$) we can think of it as describing the exchange of a string with mass given by $\alpha' M^2 = 4n$. As explained in [13] the importance of single pomeron exchange lies in the fact that it dominates scattering amplitudes in the Regge regime both in QCD for $N_c \to \infty$ and in string theory.

The information contained within the pomeron vertex is only that of the two physical states
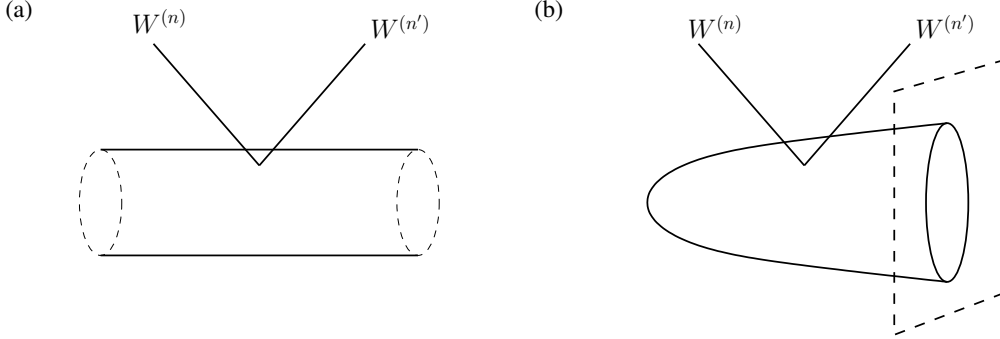
Figure 2: (a) The pomeron vertex operator given by two physical vertices W^n and $W^{n'}$ on the infinite cylinder. (b) The addition of a boundary gives the process in which these states interact with a D-brane.

which produce the exchanged pomeron and the restriction of our kinematic invariants to the Regge limit, as such this vertex may be used to generate many amplitudes of interest by insertion onto the appropriate world-sheet. In this instance we are interested in a world-sheet with a single boundary and boundary conditions which correspond to the presence of a Dp-brane, indicated in figure 2(b). This can be easily done using boundary state methods which were introduced in [21] and are reviewed in [22]. These methods are extremely powerful for dealing with problems involving string interactions with D-branes [23, 24] and for our purposes we can emulate the effects of the boundary state using the doubling trick, but further applications using the same vertex can be carried out systematically by bearing these facts in mind.

We will illustrate the use of the pomeron vertex operator with some specific examples before proceeding to the general case. If we label by (n, n') the amplitude in which a state of mass $\alpha' M_1^2 = 4n$ undergoes a transition to a state of mass $\alpha' M_2^2 = 4n'$ then the examples we shall consider will be $(0, n')$ and $(1, n')$. At the end of this section we will generalize our results to the case (n, n').

5.1 Graviton-massive state transitions

In this case we examine the processes in which a massless state is excited to another of arbitrary mass. The physical polarizations for massive states will be written as tensor products of their holomorphic and antiholomorphic components,

$$\epsilon_{\mu_1...\mu_n\alpha\,\nu_1...\nu_n\beta} = \varepsilon_{\mu_1...\mu_n\alpha} \otimes \widetilde{\varepsilon}_{\nu_1...\nu_n\beta}, \tag{36}$$

$$G_{\mu_1...\mu_n\alpha\,\nu_1...\nu_n\beta} = \mathcal{G}_{\mu_1...\mu_n\alpha} \otimes \widetilde{\mathcal{G}}_{\nu_1...\nu_n\beta}. \tag{37}$$

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Our first task will be to determine the OPE of the vertex operators $W_{(0,0)}^{(0)}(k_1, z_1, \bar{z}_1)$ and $W_{(-1,-1)}^{(n)}(k_2, z_2, \bar{z}_2)$ as $z_1 \to z_2$ and $\bar{z}_1 \to \bar{z}_2$; this task may seem daunting at first due to the large number of possible contractions, but as we shall see it is possible to immediately identify which contractions will end up giving the leading order contribution to this amplitude. In this particular computation our vertex operators are

$$
\begin{aligned}
W_{(0,0)}^{(0)}(k_1, z_1, \bar{z}_1) &= -\epsilon_{\mu\nu} V_0^\mu(k_1, z_1) \widetilde{V}_0^\nu(k_1, \bar{z}_1), \\
V_0^\mu(k_1, z_1) &= \frac{1}{\sqrt{2\alpha'}} \left(i\partial X^\mu(z_1) + 2\alpha' k_1 \cdot \psi(z_1) \psi^\mu(z_1) \right) e^{ik_1 \cdot X(z_1)},
\end{aligned} \tag{38}
$$

and

$$
\begin{aligned}
W_{(-1,-1)}^{(n')}(k_2, z_2, \bar{z}_2) &= \mathcal{G}_{\rho_1 \dots \rho_{n'}\sigma} \widetilde{\mathcal{G}}_{\lambda_1 \dots \lambda_{n'}\gamma} V_{-1}^{\rho_1 \dots \rho_{n'}\sigma}(k_2, z_2) \widetilde{V}_{-1}^{\lambda_1 \dots \lambda_{n'}\gamma}(k_2, \bar{z}_2), \\
V_{-1}^{\rho_1 \dots \rho_{n'}\sigma}(k_2, z_2) &= \frac{1}{\sqrt{n'!}} \left(\frac{i}{\sqrt{2\alpha'}} \right)^{n'} e^{-\varphi(z_2)} \prod_{i=1}^{n'} \partial X^{\rho_i} \psi^\sigma e^{ik_2 \cdot X(z_2)}.
\end{aligned} \tag{39}
$$

In deriving the pomeron vertex operator we consider first the insertion of the above two closed string vertices onto a world-sheet with the topology of the Riemann sphere. We need not consider the contractions between holomorphic and antiholomorphic operators until the resulting effective vertex is inserted onto a world-sheet with the topology of the disc. As such we can write the resulting OPE in terms of the world-sheet separation $w = z_1 - z_2$ and the point $z = \frac{z_1 + z_2}{2}$ which then takes the following form

$$
W_{(0,0)}^{(0)}\left(k_1, z + \tfrac{w}{2}, \bar{z} + \tfrac{\bar{w}}{2}\right) W_{(-1,-1)}^{(n')}\left(k_2, z - \tfrac{w}{2}, \bar{z} - \tfrac{\bar{w}}{2}\right) \sim -|w|^{-\alpha' \frac{t}{2} + 2n'} \mathcal{O}(z, w) \widetilde{\mathcal{O}}(\bar{z}, \bar{w}). \tag{40}
$$

It is simple to check that the operators $\mathcal{O}$ and $\widetilde{\mathcal{O}}$ are polynomials of at most degree $(n'+1)$ in w^{-1} and $\bar{w}^{-1}$ respectively, with an exponential factor contributing terms subleading in the small w limit, that is

$$
\mathcal{O}(z, w) = e^{i\frac{1}{2}(k_1 - k_2)\cdot \partial X(z) w} \sum_{p=1}^{n'+1} \frac{\mathcal{O}_p(z)}{w^p}, \quad \widetilde{\mathcal{O}}(\bar{z}, \bar{w}) = e^{i\frac{1}{2}(k_1 - k_2)\cdot \bar{\partial} X(\bar{z}) \bar{w}} \sum_{q=1}^{n'+1} \frac{\widetilde{\mathcal{O}}_q(\bar{z})}{\bar{w}^q}. \tag{41}
$$

In the high energy limit it is necessary to retain these particular subleading terms in the exponential, as we will see, because contractions between $k_1 \cdot \partial X$ and $k_1 \cdot \bar{\partial} X$ will generate factors of s meaning that these terms cannot be neglected for $|w|^2 \sim (\alpha' s)^{-1}$. It is in fact these terms which will generate the Regge behaviour that we expect.

The momentum exchanged between the string and the brane may be written as $q = p_1 + p_2 = 2(k_1 + k_2)$, hence $-q^2 = t$, and it is also useful to define a vector $\tilde{q} = 2(k_1 - k_2)$. In the

expansions given by equation (41) it will be the most singular terms which will dominate in the pomeron vertex operator, this operator being obtained by the integration of w in the OPE (40) over the complex plane, and this procedure will in general result in an integral for each of these terms of the form

$$\int_{\mathbb{C}} d^2w \, |w|^{-\alpha'\frac{t}{2}-2} e^{i\frac{\tilde{q}}{4}\cdot\partial X(z)w} e^{i\frac{\tilde{q}}{4}\cdot\bar{\partial}X(\bar{z})\bar{w}}. \tag{42}$$

The integration of (42) can be done by introducing new variables $u = \frac{\tilde{q}}{4} \cdot \partial X(z) \, w$, $\bar{u} = \frac{\tilde{q}}{4} \cdot \bar{\partial}X(\bar{z}) \, \bar{w}$,

$$e^{-i\pi\alpha'\frac{t}{4}} \int_{\mathbb{C}} d^2u \, |u|^{-\alpha'\frac{t}{2}-2} \, e^{i(u+\bar{u})} \left(i\frac{\tilde{q}}{4} \cdot \partial X(z) \right)^{\alpha'\frac{t}{4}} \left(i\frac{\tilde{q}}{4} \cdot \bar{\partial}X(\bar{z}) \right)^{\alpha'\frac{t}{4}}. \tag{43}$$

Then we integrate over the positions $u = re^{i\theta}$ using the following integrals:

$$\int_0^{2\pi} d\theta \, e^{-2ir\cos\theta} = 2\pi \, J_0(2r), \tag{44}$$

$$\int_0^\infty dr \, r^a \, J_0(2r) = \frac{1}{2}\frac{\Gamma\left(\frac{1+a}{2}\right)}{\Gamma\left(\frac{1-a}{2}\right)}, \tag{45}$$

and we get

$$\int_{\mathbb{C}} d^2w \, |w|^{-\alpha'\frac{t}{2}-2} e^{i\frac{\tilde{q}}{4}\cdot\partial X(z)w} e^{i\frac{\tilde{q}}{4}\cdot\bar{\partial}X(\bar{z})\bar{w}} = \Pi(t) \left(i\frac{\tilde{q}}{4} \cdot \partial X(z) \right)^{\alpha'\frac{t}{4}} \left(i\frac{\tilde{q}}{4} \cdot \bar{\partial}X(\bar{z}) \right)^{\alpha'\frac{t}{4}}, \tag{46}$$

where $\Pi(t)$ is commonly referred to as the pomeron propagator [13, 14] and is given by the following

$$\Pi(t) = 2\pi \frac{\Gamma\left(-\alpha'\frac{t}{4}\right)}{\Gamma\left(1+\alpha'\frac{t}{4}\right)} e^{-i\pi\alpha'\frac{t}{4}}. \tag{47}$$

The two-point function is reduced to a one-point function of the effective pomeron vertex on the disc.

From what we have discussed so far, one can conclude that to leading order in energy the pomeron vertex operator should take the following form

$$\int_{\mathbb{C}} d^2w \, W_{(0,0)}^{(0)}(k_1, z + \tfrac{w}{2}, \bar{z} + \tfrac{\bar{w}}{2}) W_{(-1,-1)}^{(n')}(k_2, z - \tfrac{w}{2}, \bar{z} - \tfrac{\bar{w}}{2}) \sim -K_{0,n'}(q, \epsilon, G)\Pi(t)\mathcal{O}(z)\widetilde{\mathcal{O}}(\bar{z}), \tag{48}$$

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where we have the pomeron propagator, the normal-ordered operators

$$\mathcal{O}(z) = \sqrt{2\alpha'}\left(i\frac{\tilde{q}}{4}\cdot\partial X\right)^{\alpha'\frac{t}{4}}k_1\cdot\psi e^{-\varphi}e^{i\frac{q}{2}\cdot X}, \tag{49a}$$

$$\widetilde{\mathcal{O}}(\bar{z}) = \sqrt{2\alpha'}\left(i\frac{\tilde{q}}{4}\cdot\bar{\partial}X\right)^{\alpha'\frac{t}{4}}k_1\cdot\widetilde{\psi}e^{-\widetilde{\varphi}}e^{i\frac{q}{2}\cdot X}, \tag{49b}$$

and a kinematic function, dependent upon the polarisation tensors and the momentum transferred from the string to the brane,

$$K_{0,n'}(q,\epsilon,G) = \frac{1}{n'!}\left(\frac{\alpha'}{2}\right)^{n'}(k_1)^{n'}\cdot\mathcal{G}_0\cdot\varepsilon_0\cdot(k_2)^n\,(k_1)^{n'}\cdot\widetilde{\mathcal{G}}_0\cdot\widetilde{\varepsilon}_0\cdot(k_2)^0. \tag{50}$$

Here we have introduced for later convenience the notation

$$(k_1)^{n'}\cdot\mathcal{G}_a\cdot\varepsilon_a\cdot(k_2)^n = 2^{n+n'-a-b}\left(\prod_{i=1}^{a}\eta^{\rho_i\mu_i}\right)\left(\prod_{j=a+1}^{n'}k_1^{\rho_j}\right)\left(\prod_{k=a+1}^{n}k_2^{\mu_k}\right)\mathcal{G}_{\rho_1...\rho_{n'},\sigma},\eta^{\sigma\alpha}\varepsilon_{\mu_1...\mu_n\alpha} \tag{51}$$

where $a \in \{0,\ldots,\min\{n,n'\}\}$ will count the number of contractions between $\mathcal{G}$ and ε in addition to that arising from the fermionic fields, and an analogous expression holds for the polarisation of the antiholomorphic components. In (51) products of the form $\prod_{i=1}^{0}$ should be replaced by unity. This notation will prove useful since the polarisation tensors for all states on the leading Regge trajectory are symmetric in all holomorphic indices and symmetric in all antiholomorphic indices, as a result the order of contractions with these indices is immaterial; all we need do is keep track of how many factors of k_1 are contracted with $\mathcal{G}$, $\widetilde{\mathcal{G}}$ and how many factors of k_2 are contracted with ε, $\widetilde{\varepsilon}$. Furthermore, due to the requirement that longitudinal polarisations vanish we find that we may replace all occurrences of k_1 and k_2 in equation (51) and its antiholomorphic partner with the transferred momentum $q = 2(k_1 + k_2)$. In this case we can substitute the expression in equation (51) with

$$q^{n'}\cdot\mathcal{G}_a\cdot\varepsilon_a\cdot q^n = \left(\prod_{i=1}^{a}\eta^{\rho_i\mu_i}\right)\left(\prod_{j=a+1}^{n'}q^{\rho_j}\right)\left(\prod_{k=a+1}^{n}q^{\mu_k}\right)\mathcal{G}_{\rho_1...\rho_{n'}\sigma}\eta^{\sigma\alpha}\varepsilon_{\mu_1...\mu_n\alpha}. \tag{52}$$

The one-point function of the pomeron vertex is given by the contraction of the operators $\mathcal{O}(z)$, $\widetilde{\mathcal{O}}(\bar{z})$ in (49). To leading order in s, discarding the masses, we obtain

$$K_{0,n'}(q,\epsilon,G)\int_{D_2}\frac{dzd\bar{z}}{V_{CKG}}\Pi(t)\langle:\mathcal{O}(z)::\widetilde{\mathcal{O}}(\bar{z}):\rangle_{D_2} \sim K_{0,n'}(q,\epsilon,G)(\alpha's)^{\alpha'\frac{t}{4}+1}$$

$$\times\frac{\Pi(t)}{2\pi}\Gamma\left(1+\alpha'\frac{t}{4}\right). \tag{53}$$

The factor $\frac{1}{2\pi}$ arises from the ratio between the integration over the insertion point of the vertex operator and the volume of the Conformal Killing Group of the disc, $SL(2,\mathbb{R})$.

We obtain an $SL(2,\mathbb{R})$ invariant function which gives the leading high energy behaviour of the amplitude for a graviton scattering from a D-brane into a state on the leading Regge trajectory at level n',

$$A_{0,n'}(s,t) = \frac{\kappa T_p}{2} K_{0,n'}(q,\epsilon,G)\Gamma(-\alpha'\tfrac{t}{4})e^{-i\pi\alpha'\frac{t}{4}}\,(\alpha's)^{\alpha'\frac{t}{4}+1}. \tag{54}$$

5.2 Transitions from the lowest massive state

We next move on to the case of a string with mass $M_1^2 = 4/\alpha'$ interacting with a D-brane to leave a string of mass $M_2^2 = 4n'/\alpha'$. The vertex operators are now given by

$$W^{(1)}_{(0,0)}(k_1,z_1,\bar z_1) = -\varepsilon_{\mu\alpha}\widetilde\varepsilon_{\nu\beta}V_0^{\mu\alpha}(k_1,z_1)\widetilde V_0^{\nu\beta}(k_1,\bar z_1),$$

$$V_0^{\mu\alpha}(k_1,z_1) = \frac{i}{2\alpha'}\left(i\partial X^\mu\partial X^\alpha + 2\alpha'k_1\cdot\psi\psi^\alpha\partial X^\mu - i2\alpha'\partial\psi^\mu\psi^\alpha\right)e^{ik_1\cdot X}, \tag{55}$$

and

$$W^{(n')}_{(-1,-1)}(k_2,z_2,\bar z_2) = \mathcal{G}_{\rho_1\ldots\rho_{n'}\sigma}\widetilde{\mathcal{G}}_{\lambda_1\ldots\lambda_{n'}\gamma}V_{-1}^{\rho_1\ldots\rho_{n'}\sigma}(k_2,z_2)\widetilde V_{-1}^{\lambda_1\ldots\lambda_{n'}\gamma}(k_2,\bar z_2),$$

$$V_{-1}^{\rho_1\ldots\rho_{n_2}\sigma}(k_2,z_2) = \frac{1}{\sqrt{n'!}}\left(\frac{i}{\sqrt{2\alpha'}}\right)^{n'} e^{-\varphi(z_2)}\prod_{i=1}^{n'}\partial X^{\rho_i}\psi^\sigma e^{ik_2\cdot X(z_2)}. \tag{56}$$

The methods developed in the previous example need little modification in order to deal with this problem. With them we can easily determine the form of the pomeron vertex operator and it can be written in the same manner as in the $(0,n')$ case,

$$\int_{\mathbb{C}}d^2w\,W^{(1)}_{(0,0)}(k_1,z+\tfrac{w}{2},\bar z+\tfrac{\bar w}{2})W^{(n')}_{(-1,-1)}(k_2,z-\tfrac{w}{2},\bar z-\tfrac{\bar w}{2}) \sim -K_{1,n'}(q,\epsilon,G)\Pi(t)\mathcal{O}(z)\widetilde{\mathcal{O}}(\bar z), \tag{57}$$

where this time

$$K_{1,n'}(q,\epsilon,G) = \frac{1}{n'!}\left(\frac{\alpha'}{2}\right)^{n'+1}\left[q^{n'}\cdot\mathcal{G}_0\cdot\varepsilon_0\cdot q^1\,q^1\cdot\widetilde\varepsilon_0\cdot\widetilde{\mathcal{G}}_0\cdot q^{n'} - \frac{2n'}{\alpha'}q^{n'-1}\cdot\mathcal{G}_1\cdot\varepsilon_1\cdot q^0\,q^1\cdot\widetilde\varepsilon_0\cdot\widetilde{\mathcal{G}}_0\cdot q^{n'}\right.$$

$$\left. -\frac{2n'}{\alpha'}q^{n'}\cdot\mathcal{G}_0\cdot\varepsilon_0\cdot q^1\,q^0\cdot\widetilde\varepsilon_1\cdot\widetilde{\mathcal{G}}_1\cdot q^{n'-1} + \left(\frac{n'}{2\alpha'}\right)^2 q^{n'-1}\cdot\mathcal{G}_1\cdot\varepsilon_1\cdot q^0\,q^0\cdot\widetilde\varepsilon_1\cdot\widetilde{\mathcal{G}}_1\cdot q^{n'-1}\right] \tag{58}$$

and all other quantities remain as previously defined. Because of this, the resulting amplitude will be identical to that of equation (54) other than the form of the kinematic function which is

given in (58),

$$A_{1,n'}(s,t) = \frac{\kappa T_p}{2} K_{1,n'}(q,\epsilon,G)\,\Gamma(-\alpha'\tfrac{t}{4})e^{-i\pi\alpha'\frac{t}{4}}\,(\alpha's)^{\alpha'\frac{t}{4}+1}. \tag{59}$$

Note that we have taken $2\alpha'k_1 \cdot D \cdot k_1 \sim -\alpha's$ in the large s limit, neglecting a mass term of the order of unity. In the next case we will move on to masses which contribute terms of order n, n' and we reiterate that we shall be considering only those states for which the rest mass contribution to the total energy is negligible.

5.3   Transitions within the leading Regge trajectory

Here we return to our original consideration, the process in which a string in a state on the leading Regge trajectory is scattered from a Dp-brane into some other state on the leading Regge trajectory. The vertex operators are those given by equations (15) and (13), with polarisation tensors $\varepsilon \otimes \widetilde{\varepsilon}$ and $\mathcal{G} \otimes \widetilde{\mathcal{G}}$, respectively, and our methods will imitate those we have seen already. If we initially suppose n and n' to be fixed at some finite values, then by identifying the terms in the OPE of these vertices which will give the leading high energy contributions and integrating out the dependence on their separation we obtain the same general form for the pomeron vertex operator as in section 5.2,

$$\frac{1}{n!n'!}\int_{\mathbb{C}} d^2w\, W^{(n)}_{(0,0)}(k_1, z+\tfrac{w}{2}, \bar{z}+\tfrac{\bar{w}}{2})W^{(n')}_{(-1,-1)}(k_2, z-\tfrac{w}{2}, \bar{z}-\tfrac{\bar{w}}{2}) \sim -K_{n,n'}(q,\epsilon,G)\Pi(t)\mathcal{O}(z)\widetilde{\mathcal{O}}(\bar{z}),$$
$$\tag{60}$$

where now we can see the full structure of the kinematic function, which may be written as the product of a contribution from the holomorphic operators with a contribution from the antiholomorphic operators,

$$K_{n,n'}(q,\epsilon,G) = \frac{1}{n!n'!}\left(\frac{\alpha'}{2}\right)^{n+n'}\sum_{a,b=0}^{\min\{n,n'\}}\left(-\frac{\alpha'}{2}\right)^{-a-b}C_{n,n'}(a)C_{n,n'}(b)$$
$$\times\, q^{n'-a}\cdot\mathcal{G}_a\cdot\varepsilon_a\cdot q^{n-a}\,q^{n-b}\cdot\widetilde{\varepsilon}_b\cdot\widetilde{\mathcal{G}}_b\cdot q^{n'-b}. \tag{61}$$

Here $\min\{n,n'\}$ indicates the smallest value from the set $\{n,n'\}$, and the combinatorial factors $C_{n,n'}$ (one each coming from the holomorphic and antiholomorphic contractions) come from the large number of possible contractions in the OPE which lead to the same operator after taking into account the symmetry of the polarisation tensors. These functions are given by

$$C_{n,n'}(p) = \frac{n!n'!}{p!(n-p)!(n'-p)!} \tag{62}$$

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and this can be deduced in the following manner. If we consider the contribution from the holomorphic operators then $K_{n,n'}$ is determined by all possible contractions amongst

$$\varepsilon_{\mu_1...\mu_n\alpha}\mathcal{G}_{\rho_1...\rho_{n'}\sigma} : \prod_{i=1}^{n} \partial X^{\mu_i} e^{ik_1\cdot X} :: \prod_{i=1}^{n'} \partial X^{\rho_i} e^{ik_2\cdot X} : . \tag{63}$$

Since ε and $\mathcal{G}$ are symmetric in all indices these contractions can simply be labelled by the number of contractions beween ∂X^{μ_i} and ∂X^{ρ_i}, let this be a. This being the case we must count how many ways one can generate a such contractions, first one must choose a operators from a total of n possibilities for which there are $\binom{n}{a} = n!/a!(n-a)!$ different choices. Similarly we must choose a further a operators from a set of n' possibilities giving another factor of $\binom{n'}{a}$. Finally, from this set of $2a$ operators there $a!$ possible ways to contract them in pairs. The product of these numbers gives the total number of different contractions which result in a factor $\mathcal{G}_a \cdot \varepsilon_a$ and is equal to the function $C_{n,n'}(a)$.

With this new form for the kinematic factor K we are finished, the rest of the computation having already been solved in the previous two examples. The final result for the amplitude of a finite mass string scattering from a D-brane is

$$A_{n,n'}(s,t) = \frac{\kappa T_p}{2} K_{n,n'}(q,\epsilon,G)\,\Gamma\left(-\alpha'\tfrac{t}{4}\right) e^{-i\pi\alpha'\frac{t}{4}} \left(\alpha's\right)^{\alpha'\frac{t}{4}+1}. \tag{64}$$

6 Comparison with the eikonal analysis

In a recent work [10] it has been shown that tree-level amplitudes of closed strings in the background of N Dp-branes exponentiate to give the S-matrix an operator eikonal form at high energy,

$$S(s,\mathbf{b}) = e^{i2\hat{\delta}(s,\mathbf{b})}, \qquad 2\hat{\delta}(s,\mathbf{b}) = \frac{1}{2\sqrt{s}} \int_0^{2\pi} \frac{d\sigma}{2\pi} : \mathcal{A}\left(s,\mathbf{b}+\hat{\mathbf{X}}(\sigma)\right) :, \tag{65}$$

thus generalising the field theory result for the S-matrix. Here, $\mathcal{A}(s,\mathbf{b})$ can be obtained from the Regge limit of the disk amplitude for the elastic scattering of a graviton from the Dp-branes after stripping it of its dependence upon the polarisation,

$$A(s,\mathbf{q}) = \frac{\kappa T_p}{2}\Gamma\left(\alpha'\frac{q^2}{4}\right) e^{i\pi\alpha'\frac{q^2}{4}} \left(\alpha's\right)^{-\alpha'\frac{q^2}{4}+1}, \tag{66}$$

by performing a Fourier transformation from the space of transverse momenta q to that of impact parameter b,

$$\mathcal{A}(s,\mathbf{b}) = \int \frac{d^{8-p}q}{(2\pi)^{8-p}}\, A(s,\mathbf{q}) e^{i\mathbf{b}\cdot\mathbf{q}}. \tag{67}$$

In shifting the impact parameter by the string position operator $\hat{X}$, as in [4], one can take into account the finite size of the string and it is in this respect that the eikonal operator of string theory differs from the field theory eikonal. It should be possible to derive the amplitudes evaluated by direct computation in the previous sections by using the eikonal operator. One should note that in the following analysis neither $A(s, \mathbf{q})$ nor its Fourier transform $\mathcal{A}(s, \mathbf{b})$ need be specified and so these arguments are not restricted to any particular kinematic regime beyond that already assumed for the Regge limit.

The eikonal operator depends only on the bosonic modes associated to the directions transverse to the brane and therefore its action on the bosonic modes associated to the directions parallel to the brane and on the fermionic modes is trivial. This is true also for the high energy limit of the tree-level amplitudes, as emphasized in the previous sections. In this section we shall show that the matrix elements of the eikonal operator coincides precisely with the high energy limit of the tree-level string amplitudes. Once we are satisfied that this is the case we will consider the impact parameter space representation of the result (64) in the limit of large impact parameter, equivalent to small momentum transfer.

The exponential in (65) represents a resummation of the perturbative expansion and we expect that tree level diagrams arise from the linear term in this exponential; that is to say we should be able to obtain amplitudes for the high energy limit of tree level processes from matrix elements of the operator $\bar{\mathcal{A}}$ defined to be

$$\bar{\mathcal{A}} \equiv \int_0^{2\pi} \frac{d\sigma}{2\pi} : \mathcal{A}\left(s, \mathbf{b} + \mathbf{X}(\sigma)\right) := \sum_{k=0}^{\infty} \frac{1}{k!} \frac{\partial^k \mathcal{A}(s, \mathbf{b})}{\partial b^{\mu_1} \dots \partial b^{\mu_k}} \overline{X^{\mu_1} \dots X^{\mu_k}}. \tag{68}$$

To demonstrate this technique we will first reproduce the tree-level graviton to graviton scattering amplitude, before moving on to the more general case. The initial and final states representing the graviton are:

$$|i\rangle = \epsilon_{1\mu\nu} \psi^{\mu}_{-\frac{1}{2}} \widetilde{\psi}^{\nu}_{-\frac{1}{2}} |0; 0\rangle, \tag{69a}$$

$$|f\rangle = \epsilon_{2\rho\sigma} \psi^{\rho}_{-\frac{1}{2}} \widetilde{\psi}^{\sigma}_{-\frac{1}{2}} |0; 0\rangle. \tag{69b}$$

Using the commutation relations of the fermionic modes and the fact that $\bar{\mathcal{A}}$ contains only the bosonic modes, the tree-level graviton to graviton amplitude is given by

$$\langle f| \bar{\mathcal{A}} |i\rangle = \epsilon_{1\mu\nu} \epsilon_{2\rho\sigma} \eta^{\mu\rho} \eta^{\nu\sigma} \langle 0; 0| \bar{\mathcal{A}} |0; 0\rangle = \mathrm{Tr}(\epsilon_1^T \epsilon_2) \mathcal{A}(s, \mathbf{b}). \tag{70}$$

It is clear from the definition of $\mathcal{A}(s, \mathbf{b})$ that this will reproduce the expected result in q-space upon a Fourier transformation.

Now we will show that the tree-level amplitude for a state of mass $\alpha' M_1^2 = 4n$ to become a state of mass $\alpha' M_2^2 = 4n'$ after scattering from a D-brane in the high energy limit is given by

the corresponding matrix element of the operator $\bar{\mathcal{A}}$ between the initial and final states. To do this we must show that

$$\langle n'| \, \bar{\mathcal{A}} \, |n\rangle = \mathcal{A}_{n,n'}\,(s,\mathbf{b}) \tag{71}$$

where $A_{n,n'}(s,\mathbf{b})$ is the Fourier transform of equation (64). We will use the standard oscillator modes representation of the initial and final states

$$|n\rangle = \varepsilon_{\mu_1\ldots\mu_n\alpha}\,\widetilde{\varepsilon}_{\nu_1\ldots\nu_n\beta}\,\frac{1}{n!}\prod_{i=1}^{n}\alpha_{-1}^{\mu_i}\widetilde{\alpha}_{-1}^{\nu_i}\,\psi_{-\frac{1}{2}}^{\alpha}\widetilde{\psi}_{-\frac{1}{2}}^{\beta}\,|0;0\rangle\,, \tag{72a}$$

$$|n'\rangle = \mathcal{G}_{\rho_1\ldots\rho_{n'}\sigma}\,\widetilde{\mathcal{G}}_{\lambda_1\ldots\lambda_{n'}\gamma}\,\frac{1}{n'!}\prod_{j=1}^{n'}\alpha_{-1}^{\rho_j}\widetilde{\alpha}_{-1}^{\lambda_j}\psi_{-\frac{1}{2}}^{\sigma}\widetilde{\psi}_{-\frac{1}{2}}^{\gamma}\,|0;0\rangle\,. \tag{72b}$$

Proceeding as before we obtain:

$$\langle n'| \, \bar{\mathcal{A}} \, |n\rangle = K_{\rho_1\ldots\rho_{n'}\lambda_1\ldots\lambda_{n'}\mu_1\ldots\mu_n\nu_1\ldots\nu_n}\,\langle 0;0|\prod_{i=1}^{n'}\alpha_{1}^{\rho_i}\widetilde{\alpha}_{1}^{\lambda_i}\bar{\mathcal{A}}\prod_{j=1}^{n}\alpha_{-1}^{\mu_j}\widetilde{\alpha}_{-1}^{\nu_j}\,|0;0\rangle\,, \tag{73}$$

where the polarisations are contained within the tensor

$$K_{\rho_1\ldots\rho_{n'}\lambda_1\ldots\lambda_{n'}\mu_1\ldots\mu_n\nu_1\ldots\nu_n} = \frac{1}{n!n'!}\mathcal{G}_{\rho_1\ldots\rho_{n'}\sigma}\eta^{\sigma\alpha}\varepsilon_{\mu_1\ldots\mu_n\alpha}\,\widetilde{\mathcal{G}}_{\lambda_1\ldots\lambda_{n'}\gamma}\eta^{\gamma\beta}\widetilde{\varepsilon}_{\nu_1\ldots\nu_n\beta}. \tag{74}$$

To prove that the amplitudes derived in Section 5 are well described by the eikonal, we compute the values of equation (73) and compare them with results stated in equations (61) and (64).

In expanding the operator $\bar{\mathcal{A}}$ in oscillator modes it can be seen that very few terms will contribute a nonzero value to this matrix element. Since we are considering states belonging to the leading Regge trajectory, these terms can only be composed of the modes $\alpha_{\pm 1}$, $\widetilde{\alpha}_{\pm 1}$, and since they must be normal ordered there can be at most n occurrence of the operators α_1, $\widetilde{\alpha}_1$ and n' occurrences of the operators α_{-1}, $\widetilde{\alpha}_{-1}$. As a result we only expect nonzero contributions from terms of order $2|n'-n|$ through to $2(n'+n)$. Furthermore, the terms in $\bar{\mathcal{A}}$ containing k oscillators are generated by

$$\frac{1}{k!}\frac{\partial^k \mathcal{A}(s,\mathbf{b})}{\partial b^{\mu_1}\ldots\partial b^{\mu_k}}\overline{X^{\mu_1}\ldots X^{\mu_k}}, \tag{75}$$

and the invariance of the partial derivative under a change in order of differentiation will result in the oscillators being symmetric under exchange of their indices, so rather than keeping track of these indices we need only count how many ways we can generate oscillator terms of the form given above.

Using this one can deduce that we can substitute for $\bar{A}$ in eq. (73) the following quantity

$$
\sum_{a,b=0}^{\min\{n,n'\}} \left(\frac{\alpha'}{2}\right)^{n+n'-a-b} \frac{(-1)^{n+n'}}{(n-a)!(n-b)!(n'-a)!(n'-b)!} \frac{\partial^{2(n+n'-a-b)}\mathcal{A}(s,\mathbf{b})}{\partial b^{i_1}\ldots\partial b^{\ell_{n-b}}}
$$
$$
\times \alpha_{-1}^{i_1}\ldots\alpha_{-1}^{i_{n'-a}}\widetilde{\alpha}_{-1}^{j_1}\ldots\widetilde{\alpha}_{-1}^{j_{n'-b}}\alpha_1^{k_1}\ldots\alpha_1^{k_{n-a}}\widetilde{\alpha}_1^{\ell_1}\ldots\widetilde{\alpha}_1^{\ell_{n-b}}. \quad (76)
$$

This has been written such that a and b will be seen to count the number of contractions between the polarisation tensors and they take on the range of values $a, b = 0, 1, \ldots, \min\{n, n'\}$; naturally we must count how many ways these terms may be generated using the symmetry of the partial derivatives in the expansion of $\bar{A}$. The result of this substitution is the amplitude shown below

$$
\mathcal{A}_{n,n'}(s,\mathbf{b}) = K_{\rho_1\ldots\rho_{n'}\lambda_1\ldots\lambda_{n'}\mu_1\ldots\mu_n\nu_1\ldots\nu_n} \sum_{a,b=0}^{\min\{n,n'\}} (-1)^{n+n'} \left(\frac{\alpha'}{2}\right)^{n+n'-a-b} C_{n,n'}(a)C_{n,n'}(b)
$$
$$
\times \frac{\partial^{2(n+n'-a-b)}\mathcal{A}(s,\mathbf{b})}{\partial b^{i_1}\ldots\partial b^{\ell_{n-b}}} \delta^{\rho_1 i_1}\ldots\delta^{\rho_{n'-a}i_{n'-a}}\delta^{k_1\mu_1}\ldots\delta^{k_{n-a}\mu_{n-a}}\delta^{\rho_{n'-a+1}\mu_{n'-a+1}}\ldots\delta^{\rho_{n'}\mu_n}
$$
$$
\times \delta^{\lambda_1 j_1}\ldots\delta^{\lambda_{n'-b}j_{n'-b}}\delta^{\ell_1\nu_1}\ldots\delta^{\ell_{n-b}\nu_{n-b}}\delta^{\lambda_{n'-b+1}\nu_{n'-b+1}}\ldots\delta^{\lambda_{n'}\nu_n}. \quad (77)
$$

To see that this is indeed equivalent to equation (64), one can perform a Fourier transform on equation (77) to arrive at the expected result

$$
A_{n,n'}(s,\mathbf{q}) = K_{n,n'}(\mathbf{q},\epsilon,G)A(s,\mathbf{q}), \quad (78)
$$

where now the kinematic function coincides with the one given in equation (61) and $A(s,\mathbf{q})$ is as written in (66). In principle we already have a simple prescription for using the eikonal operator that would allow us to compute the one-loop correction to the above expression, this being given by the quadratic term arising from the exponential in (65); if pomeron vertex operator methods can be extended to generate such corrections as well, then it would be interesting to see whether the equivalence between these two approaches continues to hold.

The amplitude written in momentum space has a simple structure; it is neatly factored into the graviton amplitude and some modifying kinematic function which is itself simply composed of a sum over all the ways one may saturate various contractions of the tensor (74) with the momentum $\mathbf{q}$. The amplitude written in impact parameter space, as in (77), has an index structure that is much more complicated than its Fourier transform in momentum space. However, this intricate structure is greatly simplified in the limit of very large impact parameters; in such a limit the function $\mathcal{A}$ takes the form

$$
\mathcal{A}(s,\mathbf{b}) \sim s\sqrt{\pi}\frac{\Gamma\left(\frac{6-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)}\frac{R_p^{7-p}}{b^{6-p}} + i\frac{s\pi}{\Gamma\left(\frac{7-p}{2}\right)}\sqrt{\frac{\pi\alpha' s}{\ln\alpha' s}}\left(\frac{R_p}{\sqrt{2\alpha'\ln\alpha' s}}\right)^{7-p}e^{-\frac{b^2}{2\alpha'\ln\alpha' s}}, \quad (79)
$$

where to reflect the change to coordinate space we choose to express the normalisation of the amplitude in terms of the scale R_p. To be more specific we will examine impact parameters for which $b \gg R_p \gg \sqrt{2\alpha' \ln \alpha' s}$, that is, those much larger than both the effective string length and the characteristic size of the Dp-branes; this being the case, we shall ignore the imaginary part of $\mathcal{A}(s, \mathbf{b})$ and focus on the real part. The result of these considerations is that the dominant contribution to $\mathcal{A}_{n,n'}$ comes from the term with the least number of derivatives in b. Without loss of generality let us assume that $n \leq n'$ and denote the difference between the two by $\Delta = n' - n$, then this term is given by

$$
\mathcal{A}_{n,n+\Delta}(s, \mathbf{b}) \sim s\sqrt{\pi} \frac{\Gamma\left(\frac{6-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)} R_p^{7-p} K_{\rho_1 \ldots \rho_{n+\Delta} \lambda_1 \ldots \lambda_{n+\Delta} \mu_1 \ldots \mu_n \nu_1 \ldots \nu_n} \left(-\frac{\alpha'}{2}\right)^{\Delta} \left(\frac{(n+\Delta)!}{\Delta!}\right)^2
$$

$$
\times \frac{\partial^{2\Delta} b^{-(6-p)}}{\partial b^{i_1} \ldots \partial b^{j_\Delta}} \delta^{\rho_1 i_1} \ldots \delta^{\rho_\Delta i_\Delta} \delta^{\rho_{\Delta+1} \mu_1} \ldots \delta^{\rho_{n+\Delta} \mu_n} \delta^{\lambda_1 j_1} \ldots \delta^{\lambda_\Delta j_\Delta} \delta^{\lambda_{\Delta+1} \nu_1} \ldots \delta^{\lambda_{n+\Delta} \nu_n}. \quad (80)
$$

The derivative above can be decomposed into a product consisting of an overall factor $\Delta! \, b^{-(6+2\Delta-p)}$ multiplied by some tensor which may be determined from the Gegenbauer polynomials in the following way. The Gegenbauer polynomial $C_m^{(\lambda)}(x)$ is given in terms of the hypergeometric functions by

$$
C_m^{(\lambda)}(x) = \binom{m + 2\lambda - 1}{m} {}_2F_1(-m, m + 2\lambda; l + \tfrac{1}{2}; \tfrac{1}{2}(1 - x)); \quad (81)
$$

the tensor we are interested in is obtained by taking $C_{2\Delta}^{(6-p)/2}(x)$ and for each term in this polynomial we can attach the appropriate index structure by substituting $b_{i_r}/|b|$ for each factor of x, pairing the remaining indices up with Kronecker deltas and symmetrising the result. Since it will not be required for this analysis we shall make no attempt to write the generic form for this tensor and we shall instead write it as $K(\mathbf{b}, \epsilon, G)/(n!(n+\Delta)!)$ after taking the contractions with $K_{\rho_1 \ldots \rho_{n'} \lambda_1 \ldots \lambda_{n'} \mu_1 \ldots \mu_n \nu_1 \ldots \nu_n}$, this function can be thought of as a hyperspherical harmonic. Thus equation (80) becomes

$$
\mathcal{A}_{n,n+\Delta}(s, \mathbf{b}) \sim s\sqrt{\pi} \frac{\Gamma\left(\frac{6-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)} R_p^{7-p} K(\mathbf{b}, \epsilon, G) \left(-\frac{\alpha'}{2}\right)^{\Delta} \frac{(n+\Delta)!}{n!\Delta!} \frac{1}{b^{6+2\Delta-p}}. \quad (82)
$$

From this expression it is seen that these kinds of inelastic excitations of the string begin to contribute significantly to scattering processes for impact parameters $b \leq b_\Delta$ where

$$
b_\Delta^{6+2\Delta-p} = \sqrt{\pi} s \frac{\Gamma\left(\frac{6-p}{2}\right)}{\Gamma\left(\frac{7-p}{2}\right)} R_p^{7-p} \left(\frac{\alpha'}{2}\right)^{\Delta} \frac{(n+\Delta)!}{n!\Delta!}. \quad (83)
$$

The indication is that if one first considers scattering at impact parameters so large that only elastic scattering is relevant and then begins to reduce this impact parameter then the elastic channel will be gradually absorbed, first by small string transitions and by larger transitions as b is further decreased in size. However, care should be taken in drawing quantitive conclusions from the above result since it examines only a single particular inelastic channel, whereas any physical process would have many others which aren't considered by this analysis. These issues are discussed in more depth in terms of the full S-matrix in [10].

7   Conclusions

What we have seen in this work is that at high energies the small-angle scattering of bosonic string states on the leading Regge trajectory from a stack of N Dp-branes exhibits universal tree-level behaviour, provided the masses of these states remain finite. In equation (64) this behaviour is characterised by the dependence on the square of the momentum flowing parallel to the D-branes, s, which is contained entirely within the factor $(\alpha' s)^{\alpha' \frac{t}{4}+1}$. This indicates that this process is dominated by the exchange of states from the leading Regge trajectory between the string and the D-branes. The mass of the initial and final string states will determine the form of the kinematic function $K_{n,n'}(q, \epsilon, G)$ and thereby influence the dependence of the amplitude upon the transferred momentum q. We have also shown that the tree-level result (64) in the Regge regime can be reproduced by the linear term in the perturbative expansion of the eikonal operator $\hat{\delta}(s, \mathbf{b})$ determined in [10]. Since our analysis was limited to tree-level, it would be interesting to see if the agreement persists up to one-loop calculations by comparing the quadratic terms from the eikonal operator with the inelastic annulus amplitudes.

The eikonal operator should provide a complete description of the string-brane interaction at high energy. It can be expanded in a double power series of the ratios R_p/b and $\sqrt{\alpha'}/b$, the classical and string corrections respectively. In [10] the explicit form of the eikonal operator was determined to leading order in R_p/b and to all orders in $\sqrt{\alpha'}/b$. As in [4, 5], we have shown that the string corrections are neatly taken into account by a simple shift in the impact parameter for the Fourier transform of the tree-level string amplitude. From these string corrections we have seen evidence that for large impact parameters longitudinal excitations of the string as a result of interactions with the D-brane are absent at the leading order in energy. The analysis of [10] was extended to the next-to-leading order corrections in R_p/b only for the elastic scattering angle, but not for the inelastic excitations. It would be interesting to clarify the full structure of the eikonal operator and to understand how the string corrections enter at the next-to-leading order in R_p/b. Thus for this purpose it would also be useful to move one step further in the eikonal expansion, computing the one-loop amplitude for two massive states. Furthermore, it would be of interest to see whether a simple extension to the effective vertex methods employed here can

be determined which continues to yield the full high energy expression beyond tree-level.

In this paper we kept the external states fixed while taking the high energy limit and therefore we could consistently neglect their masses by demanding that these states satisfy the relation $\alpha's >> n, n'$. Since the string spectrum contains states with arbitrarily large masses one could consider this limit and allow the masses to become very large by relaxing this condition. As long as $|n - n'|$ remains much smaller than $\alpha's$ then the momentum transferred can be kept finite and the amplitudes could still demonstrate Regge behaviour at high energy. For larger differences between the mass levels we expect that the inelastic amplitudes will decay exponentially with the energy, the behaviour typical of string scattering processes at fixed angle. It would be interesting to make these observations more precise and to perform a detailed study of the amplitudes between states of the leading Regge trajectory in the limit of large masses for the external states, using where applicable both OPE methods [11–13] and saddle-point approximations [1, 2].

Acknowledgements

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